

Section 8.3 - Finding Vertical and Horizontal Asymptotes:

To find a vertical asymptote, set the denominator equal to 0 and solve for x . The answer(s) are the values that x CANNOT be. Vertical asymptotes are $x = \#$.

examples:

$$1) \frac{2x-5}{x+7}$$

$$x = -7$$

$$2) \frac{3x+19}{(x-2)(x+4)}$$

$$x = 2, -4$$

$$3) \frac{x^2+5x-9}{x^2-7x+12}$$
$$(x-4)(x-3)$$

$$x = 4, 3$$

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Finding horizontal asymptotes is all based on the degrees of the numerator and denominator...horizontal asymptotes are $y =$

If the degree is bigger on the top, there are NO horizontal asymptotes. example: $\frac{x^2}{x}$

If the degree is bigger on the bottom, the horizontal asymptote is $y = 0$. example: $\frac{x}{x^2}$

If the degrees are the same, the horizontal asymptote is the leading coefficients of each highest exponent. example: $\frac{2x}{3x}$

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examples:

4) $\frac{3x - 8}{x^2 - 5x + 6}$

bottom
 $y=0$

5) $\frac{3x^3 - 4x^2 + 7}{x^3 - 6}$

same $\frac{3}{1}$
 $y=3$

6) $\frac{x^2 + 8x - 1}{9x - 4}$

top
none

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Putting it all together:

Find the Vertical and Horizontal Asymptotes for each function:

7) $\frac{x+1}{(x-2)(x-3)}$ bot.

8) $\frac{2x}{x-6}$

9) $\frac{x^2-9}{3x-12}$

VA: $x=2, 3$
HA: $y=0$

VA: $x=6$
HA: $y=2$

VA: $x=4$
HA: none

10) $\frac{-2x+6}{x-5}$

11) $\frac{x-1}{x^2-4x+4}$
 $(x-2)(x-2)$

12) $\frac{x^3-4x+7}{2x^2-7x+6}$
 $(2x-3)(x-2)$

VA: $x=5$
HA: $y=2$

VA: $x=2$
HA: $y=0$

VA: $x=\frac{3}{2}, 2$
HA: none

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Find the vertical and horizontal asymptotes for each function:

$$1. y = \frac{(x-4)(x+3)}{x+3}$$

VA: $x = -3$
HA: none

$$2. y = \frac{(x-3)(x+1)}{x-2}$$

VA: $x = 2$
HA: none

$$3. y = \frac{2}{x+1}$$

VA: $x = -1$
HA: $y = 0$

$$4. y = \frac{4x}{x^4 + 16}$$

$x^4 = -16$

VA: none
HA: $y = 0$

$$5. y = \frac{5-x}{x^2-1}$$

VA: $x = 1, -1$
HA: $y = 0$

$$6. y = \frac{x^2-2}{x+2}$$

VA: $x = -2$
HA: none

$$7. y = \frac{x}{x(x-1)}$$

VA: $x = 0, 1$
HA: $y = 0$

$$8. y = \frac{x+3}{x^2-9}$$

VA: $x = 3, -3$
HA: $y = 0$

$$9. y = \frac{x-2}{(x+2)(x-2)}$$

$$VA: x = -2, 2$$

$$HA: y = 0$$

$$10. y = \frac{x^2-4}{x^2+4}$$

$$VA: \text{none}$$

$$HA: y = 1$$

$$11. y = \frac{x^2-25}{x-4}$$

$$VA: x = 4$$

$$HA: \text{none}$$

$$12. y = \frac{(x-2)(2x+3)}{(5x+4)(x-3)} \quad \frac{2x^2}{5x}$$

$$VA: x = -\frac{4}{5}, 3$$

$$HA: y = \frac{2}{5}$$

$$13. y = \frac{2}{x-6}$$

$$VA: x = 6$$

$$HA: y = 0$$

$$14. y = \frac{x+2}{x-4}$$

$$VA: x = 4$$

$$HA: y = 1$$

$$15. y = \frac{2x^2+3}{x^2-6} \quad x = \pm\sqrt{6}$$

$$VA: x = \pm\sqrt{6}$$

$$HA: y = 2$$

$$16. y = \frac{3x-12}{x^2-2} \quad x = \pm\sqrt{2}$$

$$VA: x = \pm\sqrt{2}$$

$$HA: y = 0$$

Section 8.3 - Finding x and y intercepts

To find the x-intercept, make the y value 0.

To find the y-intercept, make the x value 0.

examples:

$$1) y = \frac{3x-15}{x+5}$$

$$2) y = \frac{2x+18}{(x-2)(x+3)}$$

$$3) y = \frac{2x^2+9x+9}{x^2-7x+12}$$
$$(2x+3)(x+3)$$

$$\text{x-int: } \underline{(5, 0)}$$
$$\text{y-int: } \underline{(0, -3)}$$

$$\text{x-int: } \underline{(-9, 0)}$$
$$\text{y-int: } \underline{(0, -3)}$$

$$\text{x-int: } \underline{\left(-\frac{3}{2}, 0\right) \quad (-3, 0)}$$
$$\text{y-int: } \underline{(0, \frac{3}{4})}$$

Putting it all together:

Find the x-intercept, y-intercept, vertical and horizontal Asymptotes for each function:

$$4) \frac{x+4}{(x-2)(x-4)}$$

$$5) \frac{2}{x-1}$$

$$6) \frac{x^2-9}{3x-12}$$

$$\text{x-int: } \underline{(-4, 0)}$$
$$\text{y-int: } \underline{(0, \frac{1}{2})}$$
$$\text{VA: } \underline{x=2, x=4}$$
$$\text{HA: } \underline{y=0}$$

$$\text{x-int: } \underline{\text{none}}$$
$$\text{y-int: } \underline{(0, -2)}$$
$$\text{VA: } \underline{x=1}$$
$$\text{HA: } \underline{y=0}$$

$$\text{x-int: } \underline{(3, 0) \quad (-3, 0)}$$
$$\text{y-int: } \underline{(0, \frac{3}{4})}$$
$$\text{VA: } \underline{x=4}$$
$$\text{HA: } \underline{\text{none}}$$

$$7) \frac{-3x+6}{x-2}$$

$$8) \frac{9-14x}{2x+3}$$

$$9) \frac{24}{2x^2-7x+6}$$

$$(2x-3)(x-2)$$

$$\begin{aligned} \text{x-int: } & \underline{(-2, 0)} \\ \text{y-int: } & \underline{(0, -3)} \\ \text{VA: } & \underline{x=2} \\ \text{HA: } & \underline{y=-3} \end{aligned}$$

$$\begin{aligned} \text{x-int: } & \underline{\left(\frac{9}{14}, 0\right)} \\ \text{y-int: } & \underline{(0, 3)} \\ \text{VA: } & \underline{x=-\frac{3}{2}} \\ \text{HA: } & \underline{y=-7} \end{aligned}$$

$$\begin{aligned} \text{x-int: } & \underline{\text{none}} \\ \text{y-int: } & \underline{(0, 4)} \\ \text{VA: } & \underline{x=\frac{3}{2}, 2} \\ \text{HA: } & \underline{y=0} \end{aligned}$$

Homework: Worksheet 8.3 #2

Find the x-intercepts, y-intercepts, vertical asymptotes, and horizontal asymptotes for each function.

$$1) y = \frac{5x}{x^2 - 2x - 3}$$

$$(x-3)(x+1)$$

$$2) y = \frac{x-4}{(x+1)(x-2)}$$

$$3) y = \frac{x^2}{x^2 + 1}$$

$$\begin{aligned} \text{x-int} &= (0, 0) \\ \text{y-int} &= (0, 0) \\ \text{VA} &= x = 3, -1 \\ \text{HA} &= y = 0 \end{aligned}$$

$$\begin{aligned} \text{x-int} &= (4, 0) \\ \text{y-int} &= (0, 2) \\ \text{VA} &= x = -1, 2 \\ \text{HA} &= y = 0 \end{aligned}$$

$$\begin{aligned} \text{x-int} &= (0, 0) \\ \text{y-int} &= (0, 0) \\ \text{VA} &= \text{none} \\ \text{HA} &= y = 1 \end{aligned}$$

$$4) y = \frac{2}{4x^2 - 1}$$

$$(2x+1)(2x-1)$$

$$5) y = \frac{2x^2}{x^2 + 5}$$

$$6) y = \frac{2x^2}{x-8}$$

$$\begin{aligned} \text{x-int} &= \text{none} \\ \text{y-int} &= (0, -2) \\ \text{VA} &= x = -\frac{1}{2}, \frac{1}{2} \\ \text{HA} &= y = 0 \end{aligned}$$

$$\begin{aligned} \text{x-int} &= (0, 0) \\ \text{y-int} &= (0, 0) \\ \text{VA} &= \text{none} \\ \text{HA} &= y = 2 \end{aligned}$$

$$\begin{aligned} \text{x-int} &= (0, 0) \\ \text{y-int} &= (0, 0) \\ \text{VA} &= x = 8 \\ \text{HA} &= \text{none} \end{aligned}$$

$$7) y = \frac{5-20x}{4x+1}$$

$$8) y = \frac{x^2-4}{2x+3}$$

$$9) y = \frac{8x^3}{x^2-9}$$

$$\begin{aligned} \text{x-int} &= \underline{(1/4, 0)} \\ \text{y-int} &= \underline{(0, 5)} \\ \text{VA} &= \underline{x = -1/4} \\ \text{HA} &= \underline{y = -5} \end{aligned}$$

$$\begin{aligned} \text{x-int} &= \underline{(2, 0)(-2, 0)} \\ \text{y-int} &= \underline{(0, -4/3)} \\ \text{VA} &= \underline{x = -3/2} \\ \text{HA} &= \underline{\text{none}} \end{aligned}$$

$$\begin{aligned} \text{x-int} &= \underline{(0, 0)} \\ \text{y-int} &= \underline{(0, 0)} \\ \text{VA} &= \underline{x = 3, -3} \\ \text{HA} &= \underline{\text{none}} \end{aligned}$$

$$10) y = \frac{2x}{3x-1}$$

$$11) y = \frac{x+3}{x+1}$$

$$12) y = \frac{x^2-36}{x+1}$$

$$\begin{aligned} \text{x-int} &= \underline{(0, 0)} \\ \text{y-int} &= \underline{(0, 0)} \\ \text{VA} &= \underline{x = 1/3} \\ \text{HA} &= \underline{y = 2/3} \end{aligned}$$

$$\begin{aligned} \text{x-int} &= \underline{(-3, 0)} \\ \text{y-int} &= \underline{(0, 3)} \\ \text{VA} &= \underline{x = -1} \\ \text{HA} &= \underline{y = 1} \end{aligned}$$

$$\begin{aligned} \text{x-int} &= \underline{(6, 0)(-6, 0)} \\ \text{y-int} &= \underline{(0, -36)} \\ \text{VA} &= \underline{x = -1} \\ \text{HA} &= \underline{\text{none}} \end{aligned}$$

Section 8.3: Graphing Rational Functions

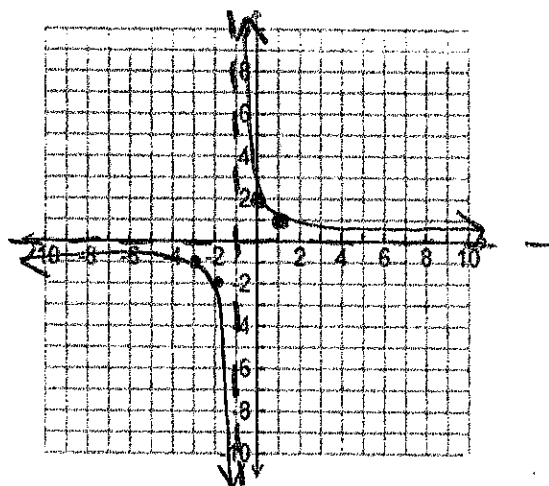
1) $y = \frac{2}{x+1}$

x-int: none

y-int: none

VA: $x = -1$

HA: $y = 0$



then use x/y chart to find additional points

x	y	x	y
1	2	-2	-2
0	2	-3	-1

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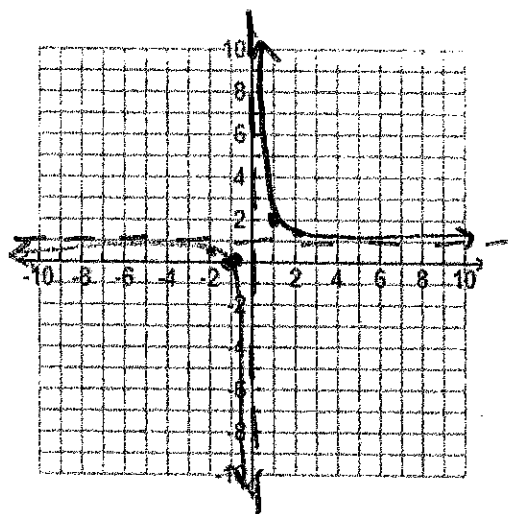
2) $y = \frac{x+1}{x}$

x-int: $(-1, 0)$

y-int: none

VA: $x = 0$

HA: $y = 1$



x/y chart

x	y	x	y
-2	1/2	1	2
1	2	2	1.5

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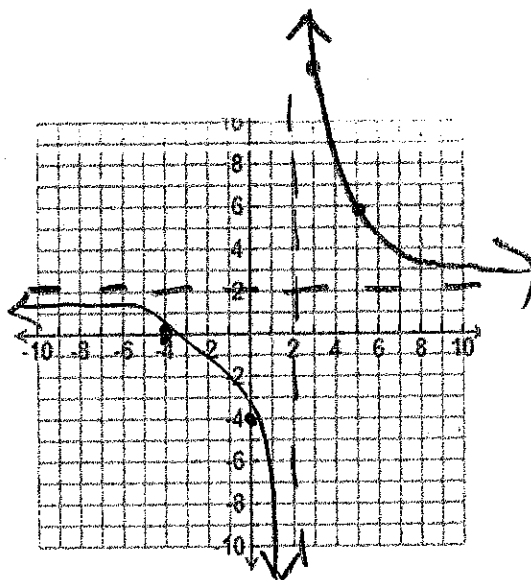
$$3) y = \frac{2x+8}{x-2}$$

$$\text{x-int: } \underline{(-4, 0)}$$

$$\text{y-int: } \underline{(0, -4)}$$

$$\text{VA: } \underline{x = 2}$$

$$\text{HA: } \underline{y = 2}$$



x/y chart

X	Y
3	14
5	6

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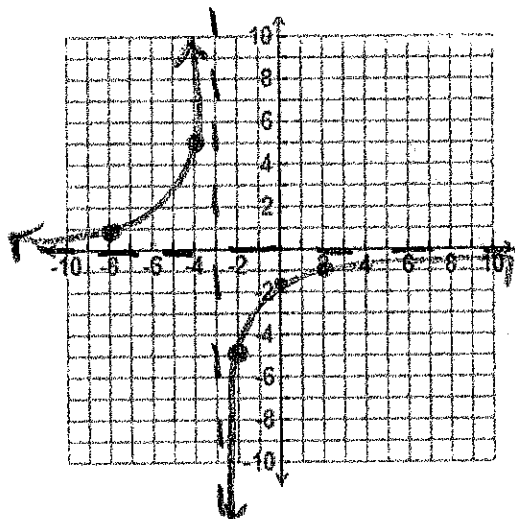
$$4) y = \frac{-5}{x+3}$$

$$\text{x-int: } \underline{\text{none}}$$

$$\text{y-int: } \underline{(0, -\frac{5}{3})}$$

$$\text{VA: } \underline{x = -3}$$

$$\text{HA: } \underline{y = 0}$$



x/y chart:

X	Y
2	-1
-2	-5

X	Y
-4	5
-8	1

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Algebra 2

8.3 Worksheet #3 - Graphing Rational Functions

Name: _____

Date: _____ Pd: _____

1.) $y = \frac{3x}{x-5}$

1.) x-int: $(0,0)$

2.) y-int: $(0,0)$

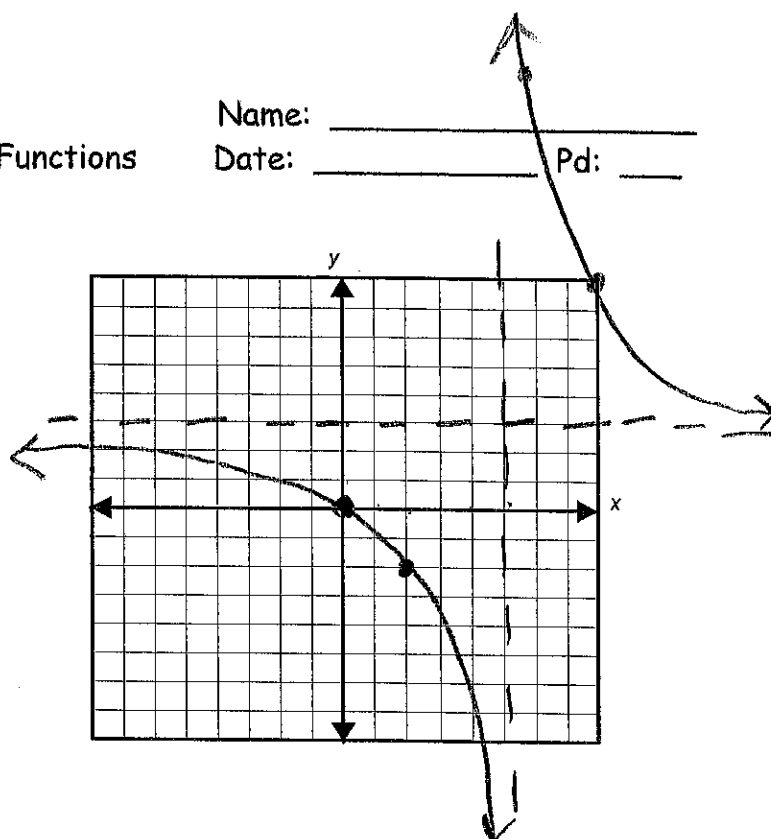
3.) VA: $x = 5$

4.) HA: $y = 3$

5.) x/y chart:

x	y
2	-2

x	y
6	18
8	8



2.) $y = \frac{2x}{x+3}$

1.) x-int: $(0,0)$

2.) y-int: $(0,0)$

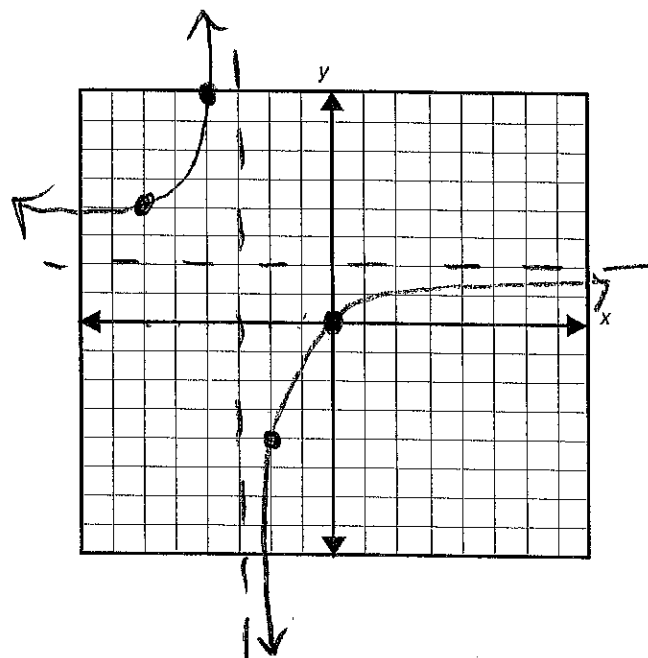
3.) VA: $x = -3$

4.) HA: $y = 2$

5.) x/y chart:

x	y
-2	-4

x	y
-4	8
-6	4



3.) $y = \frac{-6}{x-4}$

1.) x-int: none

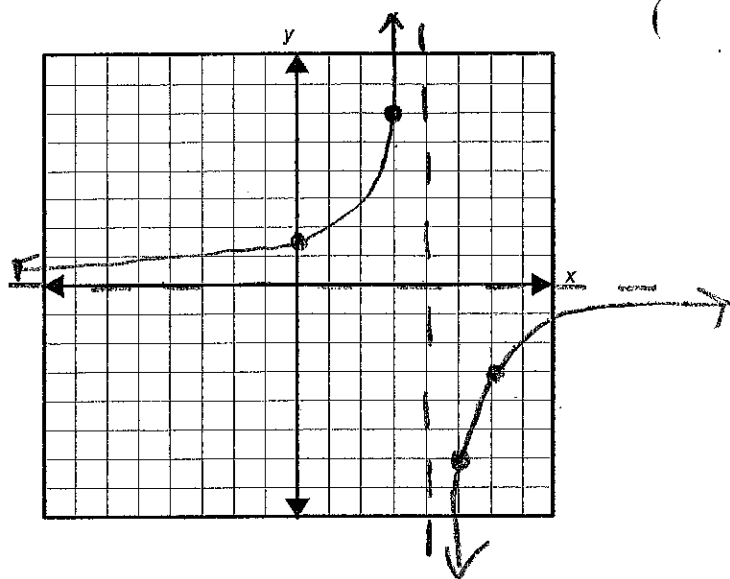
2.) y-int: (0, 1.5)

3.) VA: $x = 4$

4.) HA: $y = 0$

5.) x/y chart:

x	y	x	y
3	6	5	-6
		6	-3



4.) $y = \frac{6-x}{x+4}$

1.) x-int: (6, 0)

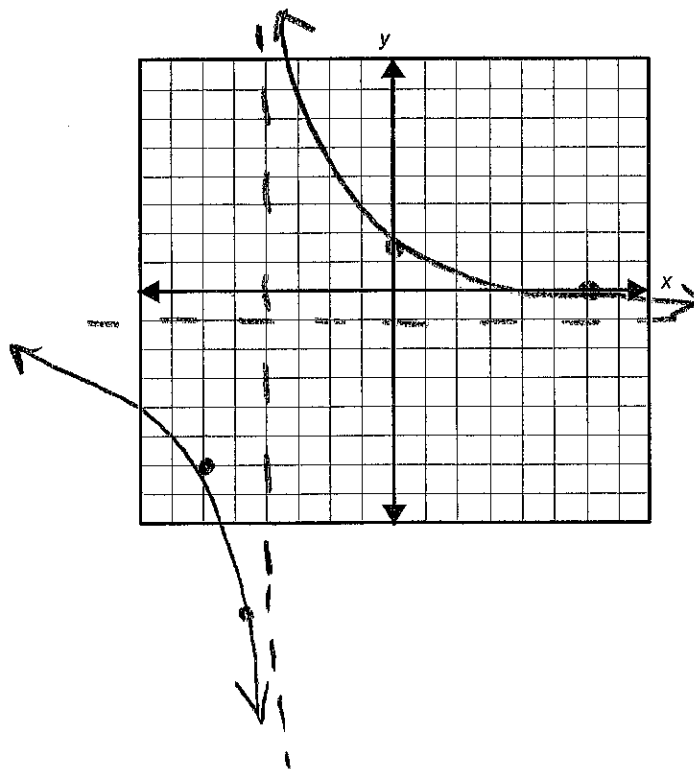
2.) y-int: (0, 1.5)

3.) VA: $x = -4$

4.) HA: $y = -1$

5.) x/y chart:

x	y
-5	-11
-6	-6
-8	



Algebra 2
8.3 Worksheet #4

Name: _____

Date: _____ Pd: _____

1.) $y = \frac{-3}{x+1}$

1.) x-int: none

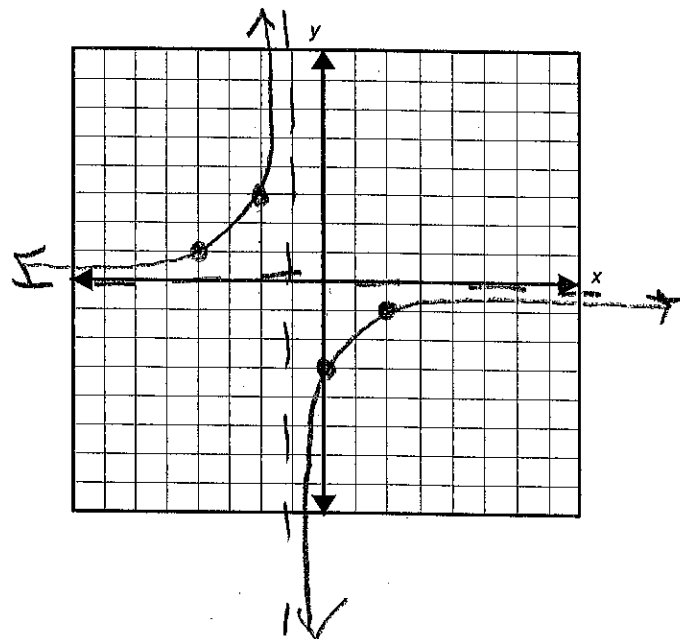
2.) y-int: (0, -3)

3.) VA: $x = -1$

4.) HA: $y = 0$

5.) x/y chart:

x	y	x	y
2	-1	-2	3
		-4	1



2.) $y = \frac{x+5}{x}$

1.) x-int: (-5, 0)

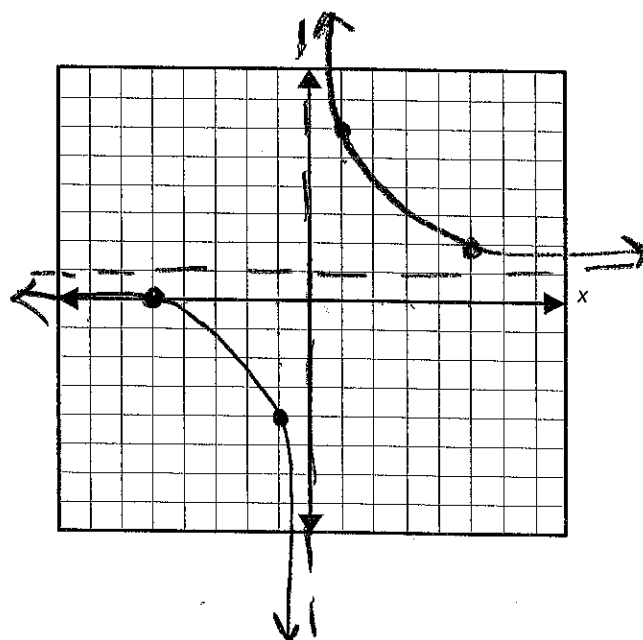
2.) y-int: none

3.) VA: $x = 0$

4.) HA: $y = 1$

5.) x/y chart:

x	y	x	y
-1	-4	1	6
		5	2



Algebra 2
8.3 Worksheet #4

Name: _____
Date: _____ Pd: _____

3.) $y = \frac{x-2}{x-1}$

1.) x-int: $(2, 0)$

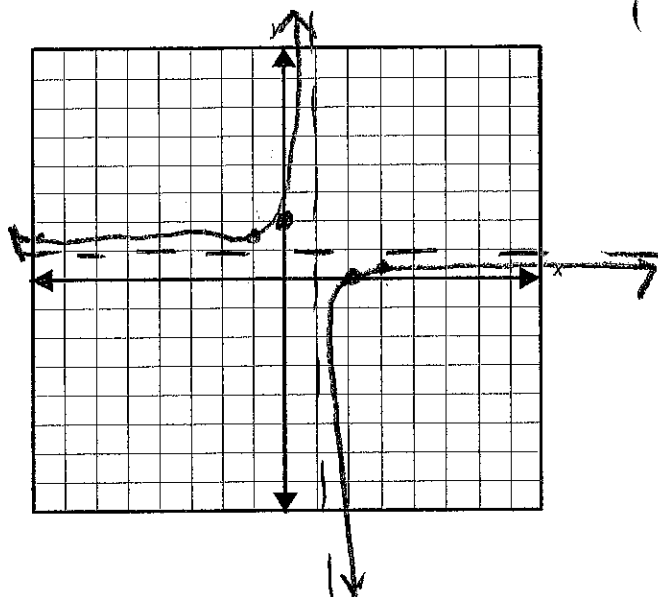
2.) y-int: $(0, 2)$

3.) VA: $x = 1$

4.) HA: $y = 1$

5.) x/y chart:

x	y
3	$1\frac{1}{2}$
-1	$3\frac{1}{2}$



4.) $y = \frac{-1}{x-4}$

1.) x-int: none

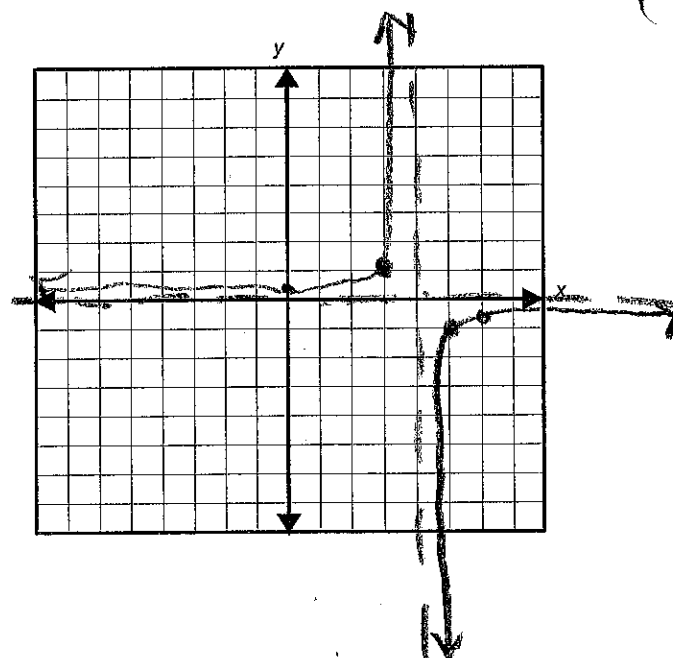
2.) y-int: $(0, -\frac{1}{4})$

3.) VA: $x = 4$

4.) HA: $y = 0$

5.) x/y chart:

x	y
3	-1
5	-1
6	$-\frac{1}{2}$



Algebra 2
8.3 Worksheet #4

Name: _____

Date: _____ Pd: _____

5.) $y = \frac{x}{x^2 - 1}$

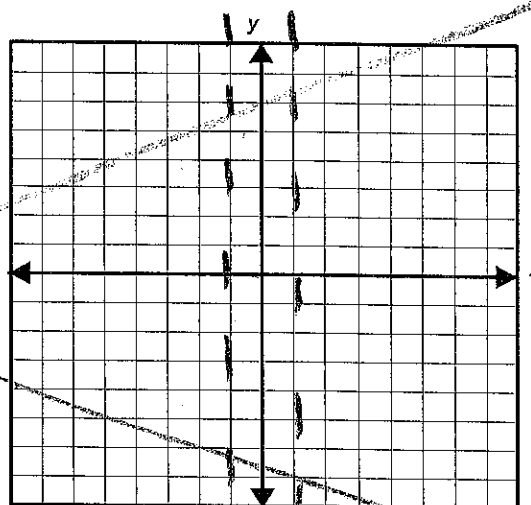
1.) x-int: (0,0)

2.) y-int: (0,0)

3.) VA: $x = 1, -1$

4.) HA: $y = 0$

5.) x/y chart:



6.) $y = \frac{2}{x}$

1.) x-int: none

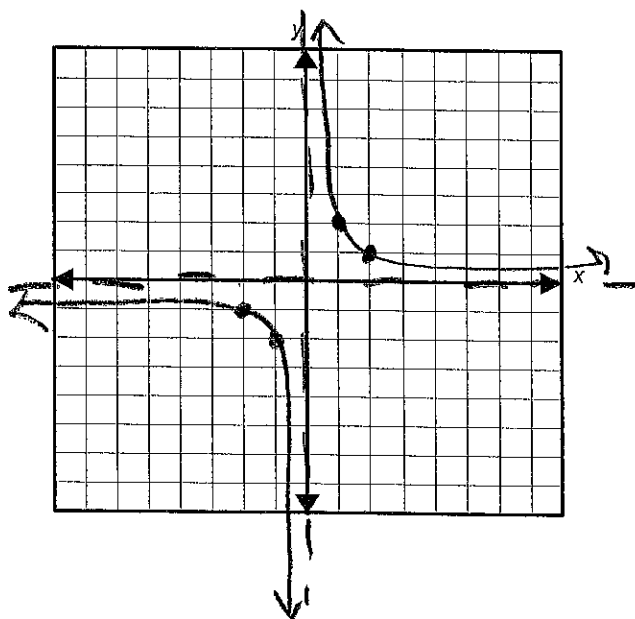
2.) y-int: none

3.) VA: $x = 0$

4.) HA: $y = 0$

5.) x/y chart:

x	y	x/y	x/y
1	2	-1	-2
2	1	-2	-1



Algebra 2
8.3 Worksheet #4

Name: _____
Date: _____ Pd: _____

7.) $y = \frac{-3x+1}{x+3}$

1.) x-int: $(\frac{1}{3}, 0)$

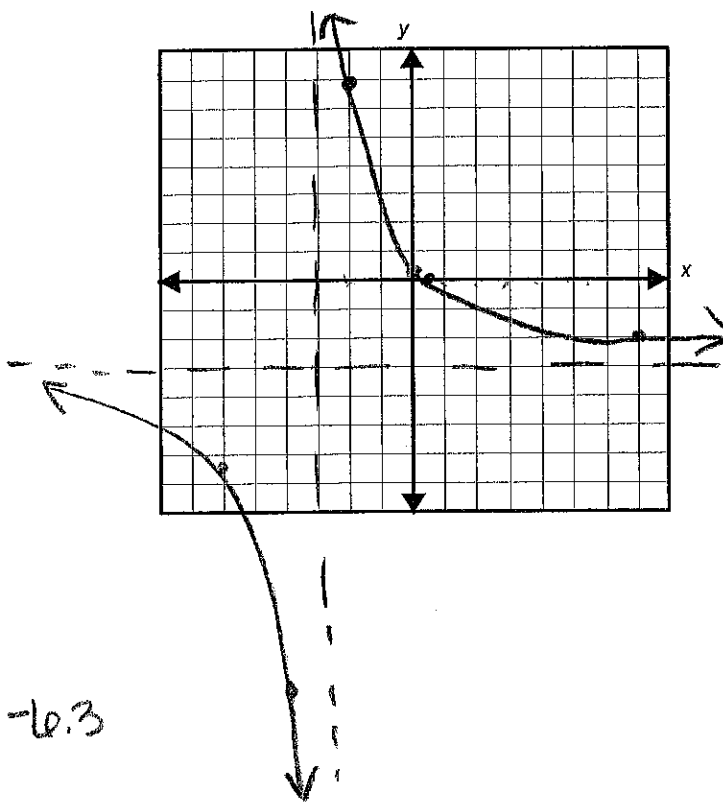
2.) y-int: $(0, \frac{1}{3})$

3.) VA: $x = -3$

4.) HA: $y = -3$

5.) x/y chart:

X	y	X	y
-2	7	-4	-13
7	-2	-6	19
			$19-3 = -6.3$



8.) $y = \frac{4x-1}{x}$

1.) x-int: $(\frac{1}{4}, 0)$

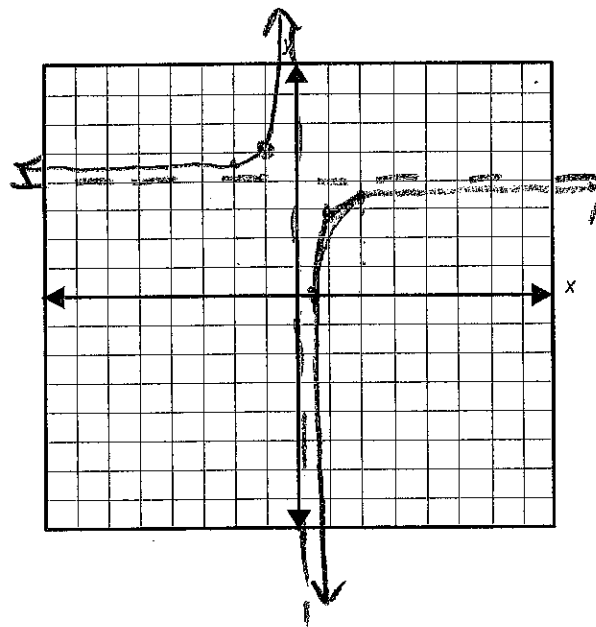
2.) y-int: none

3.) VA: $x = 0$

4.) HA: $y = 4$

5.) x/y chart:

X	y	X	y
2	7	-1	5
1	3	-2	9
			$9-4 = 4.5$



Simplify each expression.

$$1) \frac{2x^2 + 9x + 9}{x+1} \div \frac{2x^2 - 5x - 12}{x^2 - 3x - 4} =$$

$$\frac{(2x+3)(x+3)}{\cancel{x+1}} \cdot \frac{(x-4)(x+1)}{(2x+3)(x-4)}$$

$$\underline{x+3}$$

$$2) \frac{2x}{x^2 + x - 6} - \frac{3x}{x^2 - 4} =$$

$$(x+3)(x-2) \mid (x+2)(x-2)$$

$$\text{LCD: } (x-2)(x+3)(x+2)$$

$$\frac{-x^2 - 5x}{(x-2)(x+3)(x+2)}$$

$$2x(x+2) - 3x(x+3)$$

$$2x^2 + 4x - 3x^2 - 9x$$

$$3) \frac{x^2 - x - 6}{x^2 + 6x + 9} \cdot \frac{x+3}{x^2 - 4} =$$

$$\frac{(x-3)(x+2)}{(x+3)(x+3)} \cdot \frac{(x+3)}{(x+2)(x-2)}$$

$$\underline{\frac{x-3}{(x+3)(x-2)}}$$

$$4) \frac{2x}{x^2 + 5x + 4} + \frac{3x}{3x+3} =$$

$$(x+4)(x+1) \mid 3(x+1)$$

$$\text{LCD: } 3(x+1)(x+4)$$

$$\underline{\frac{3x^2 + 18x}{3(x+1)(x+4)}}$$

$$2x \cdot 3 + 3x(x+4)$$

$$6x + 3x^2 + 12x$$

Solve each of the following equations. Make sure to find the restricted values.

5) $\frac{2x-3}{5} = \frac{2x-5}{6}$

LCD: $5 \cdot 6$

$x \neq$ no restrictions

$$6(2x-3) = 5(2x-5)$$

$$12x - 18 = 10x - 25$$

$$2x - 18 = -25$$

$$2x = -7 \quad x = -\frac{7}{2}$$

$x =$ $-\frac{7}{2}$

6) $\frac{4}{x-3} = \frac{2}{x+1} + \frac{16}{x^2-2x-3}$

$x \neq$ $3, -1$

$$4(x+1) = 2(x-3) + 16$$

$$4x + 4 = 2x - 6 + 16$$

$$4x + 4 = 2x + 10 \quad 2x = 6$$

$$2x + 4 = 10$$

$$x = 3$$

$x =$ no solution

Find the vertical and horizontal asymptotes for each function.

7) $y = \frac{3x+1}{x-5}$

8) $y = \frac{x+1}{3x^2-11x+6}$

$$(3x-2)(x-3)$$

9) $y = \frac{3x^3-4}{4x+1}$

VA: $x = 5$

VA: $x = \frac{2}{3}, 3$

VA: $x = -\frac{1}{4}$

HA: $y = 3$

HA: $y = 0$

HA: none

Sketch the graph of each rational function. Show all work.

10) $y = \frac{3}{x}$

x-int: none

y-int: none

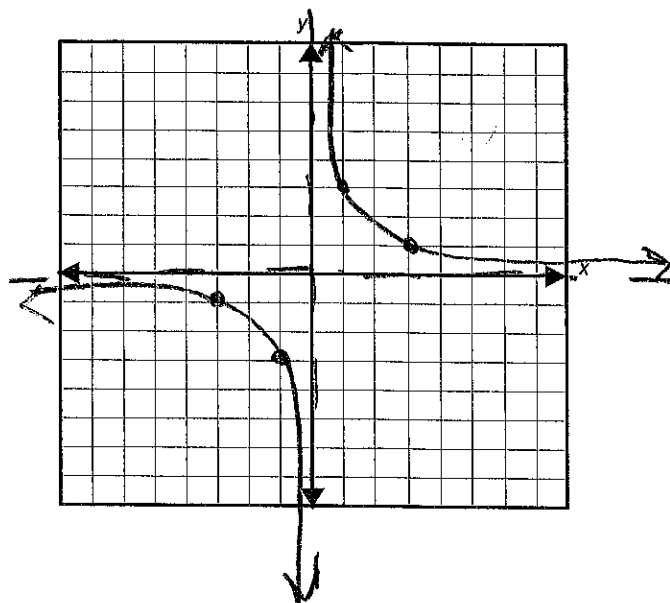
VA: $x = 0$

HA: $y = 0$

x/y chart:

x	y
1	3
3	1

x	y
-1	-3
-3	-1



11) $y = \frac{x-5}{x+1}$

x-int: (5, 0)

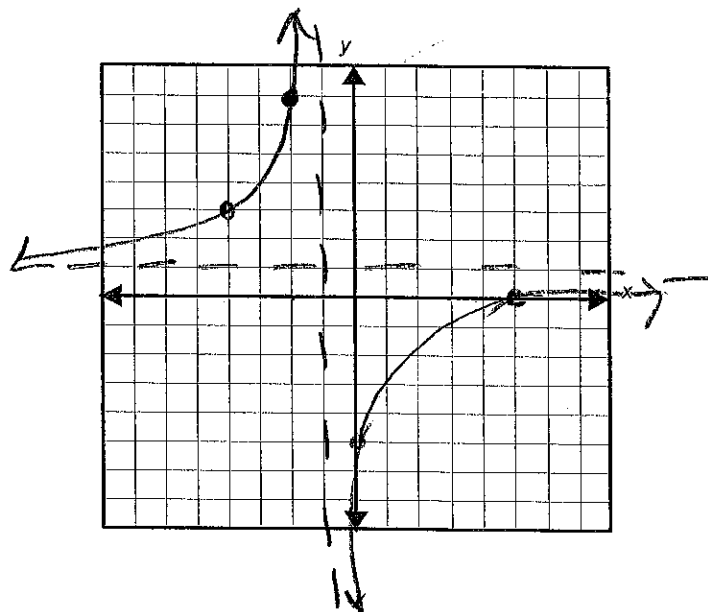
y-int: (0, -5)

VA: $x = -1$

HA: $y = 1$

x/y chart:

x	y
-2	7
-4	3



12) $y = \frac{4x}{2x-6}$

x-int: (0,0)

y-int: (0,0)

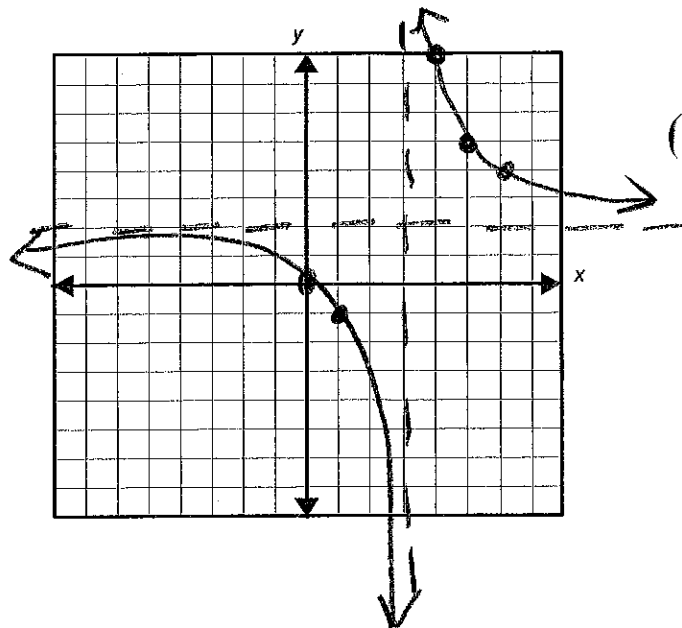
VA: $x=3$

HA: $y=2$

x/y chart:

x	y
1	-1

x	y
4	8
5	5
6	4



13) $y = \frac{x^2 - 4}{x}$

x-int: (2,0)(-2,0)

y-int: none

VA: $x=0$

HA: none

x/y chart:

x	y
1	-3
4	3
8	7.5

x	y
-1	3
-4	-3
-8	-7.5

